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I dedicate this work to my grandparents:

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Abstract

Recent developments in instrumentation have increased the use of contact mechanical testing techniques (“nanoindentation”) for the examination of local mechanical responses. These techniques are ideal for testing tissue biomechanical responses on the scale of the tissue ultrastructure. However, features of the mechanical response of tissues, such as time-dependence and inhomogeneity, add complexity to the analysis of indentation testing. The current work is aimed at promoting understanding of mechanics of biological tissues at the level of the ultrastructure. Engineering materials with known composition and properties are used as model materials for exploring the indentation mechanical behavior of time-dependent and inhomogeneous systems. Modeling techniques, both analytical and computational, are used to understand the fundamental mechanical behavior of biological composite material systems at ultrastructural length-scales. Models of mineralized tissue behavior at small scales are developed for both homogeneous and indentation loading. After establishing the mechanics framework for investigating these material types, indentation experiments are performed on biological materials and mechanical properties are extracted using the models that have been developed. Techniques developed in the current work are used to examine variations in mechanical response of healing bone as a function of distance from the dental implant interface and as a function of healing time since implantation. Relative contributions of mineral content and local mineral network structure on variations in elastic modulus were examined.

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